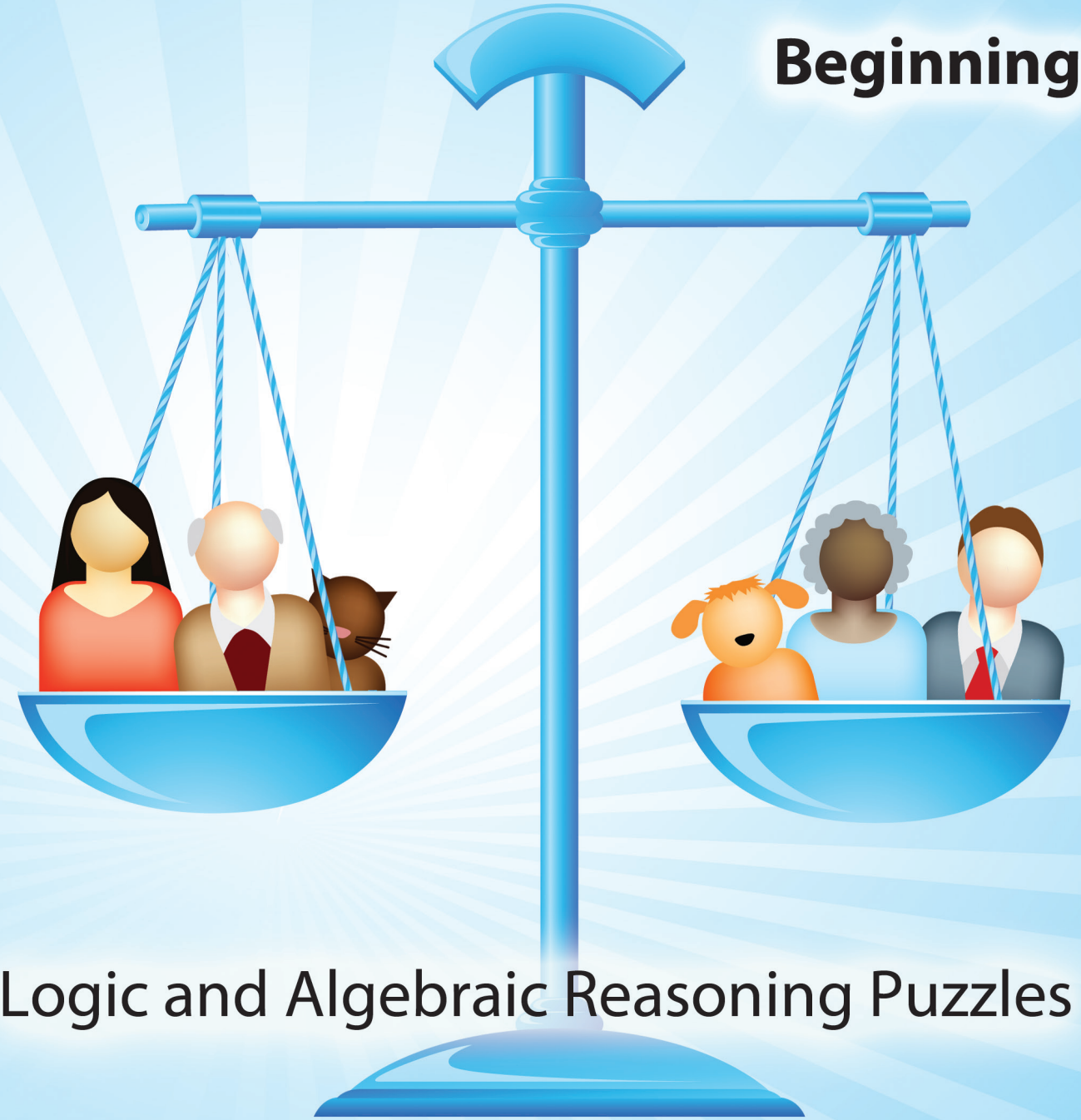
The Critical Thinking Co.™
Empower the mind!

GRADES 2–6
06760BEP

Balance Benders™

Award Winning!

Beginning



Logic and Algebraic Reasoning Puzzles

Robert Femiano

Table of Contents

The Value of This Book

Move over Sudoku, here come *Balance Benders*™! You can use these books as quick, fun, logic problems or as stepping-stones to success in algebra. Students develop deductive thinking and pre-algebra skills as they solve balance puzzles that are more fun and addictive than Sudoku puzzles! Students must analyze each balance to identify the clues, and then synthesize the information to solve the puzzle. Try one—and then try to stop!

Teaching Suggestions

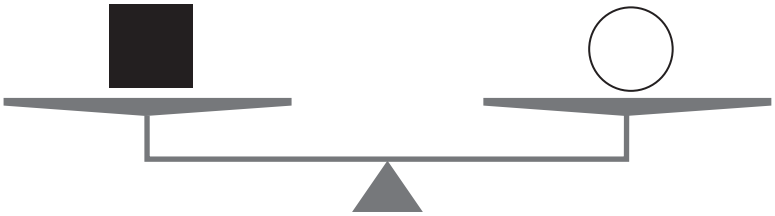
Before beginning the puzzles with your students, review the 10 Balance Tips (Algebra Concepts) listed on pages 41 and 42. Each puzzle's solution involves one or more of these basic concepts. Next, work the first puzzle with the students. After you work the puzzle and identify the correct answers, reread the 10 Balance Tips with the students to make sure they are familiar with all of them. Continue to work through the puzzles with the students until they demonstrate the ability to solve puzzles independently.

Once the students are working independently they might occasionally be stumped by a puzzle. If this happens, you can either ask them to return to the puzzle later after they take a break, or offer a clue using the Balance Tips found in the solutions on pages 43-46. It often helps to remind students that the joy of puzzles is being puzzled. Do your best to keep these puzzles fun, and remember that it is just as important to praise perseverance as it is to praise the correct answer.

About the Author

A longtime puzzle fan, Robert Femiano is a Seattle public school elementary educator and has been for most of his 34-year teaching career. For more than a decade of this time, he was also adjunct faculty at Seattle Pacific University conducting math methods courses. Publications include *Algebraic Problem Solving in the Primary Grades* in the National Council for Teachers of Mathematics peer-reviewed journal and *Quick Thinks Math* books and software by The Critical Thinking Co.™. In 2002, he won the highest honor in education, the Presidential Award for Excellence in Mathematics and Science Teaching.

Balance Benders™



Circle the three answers that will always be true.

a. ○ = ■

b. ○ ○ = ■ ■ ■

c. ■ ○ = ○ ○

d. ■ ■ = ○ ○ ○

e. ○ ○ = ■ ■

f. ○ ■ = ■ ■ ○

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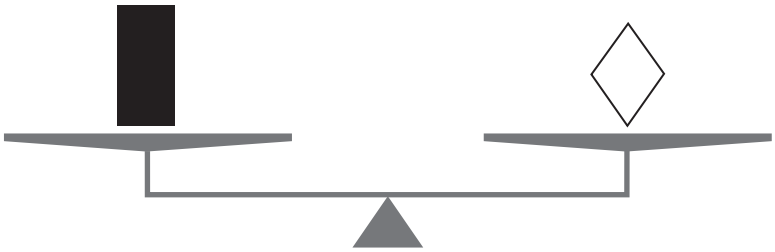

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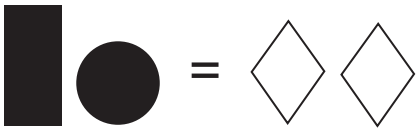
a. 

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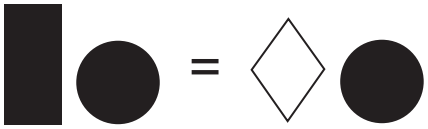
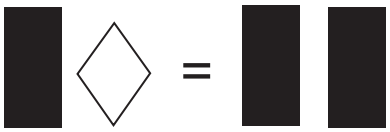
c. 

d. 

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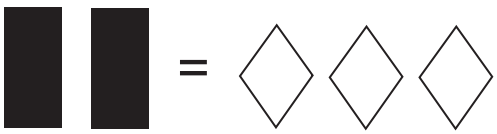
f. 

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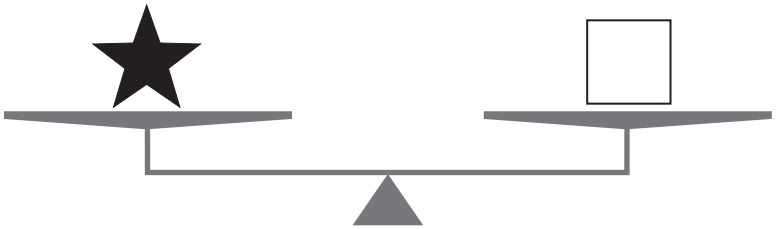

Circle the three answers that will always be true.

a. ★★★ = □□

b. □★ = □□

c. ★□ = ★★★

d. ★□★ = □★

e. ★★ = □★

f. ◆★ = □◆

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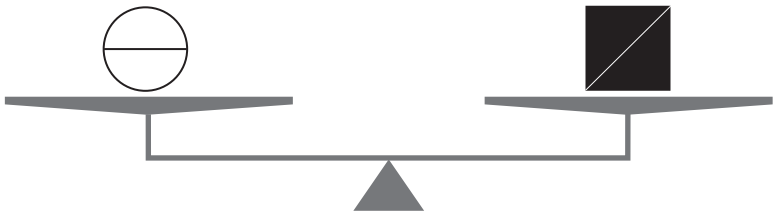

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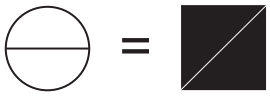
a. 

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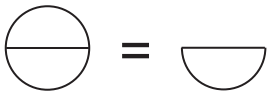
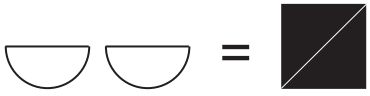
c. 

d. 

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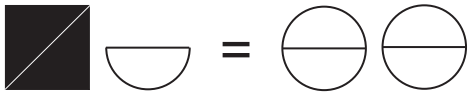
f. 

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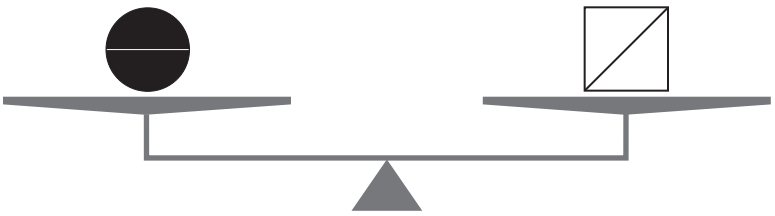


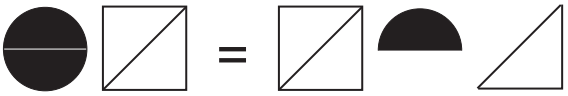
Circle the three answers that will always be true.

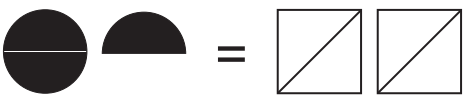
a. 

b. 

c. 

d. 

e. 

f. 

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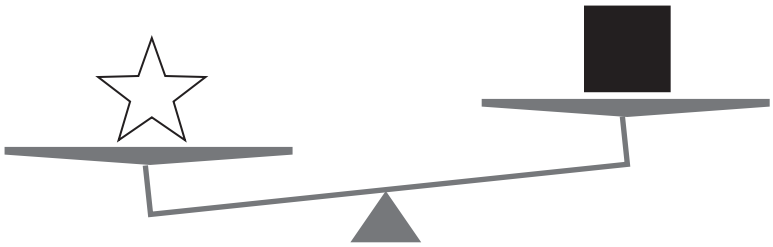


Circle the three answers that will always be true.

a. ☆ = ■

b. ■ > ☆

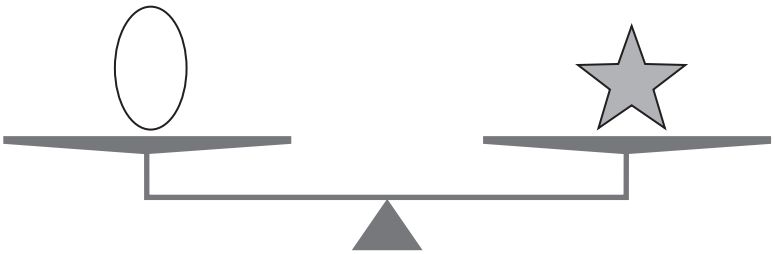
c. ☆ > ■

d. ■ < ☆

e. ☆ < ■

f. ■ ■ < ☆ ☆

Balance Benders™



Circle the three answers that will always be true.

a. ○ > ★

b. ★ > ○

c. ★ ★ = ○ ○

d. ■ ♦ ★ = ♦ ○ ■

e. ★ ★ ★ > ○ ○

f. ○ < ★

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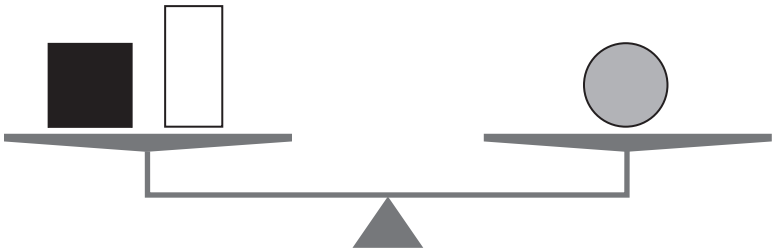

Circle the three answers that will always be true.

a. ■ < ▯

b. ● > ▯ ▯

c. ■ < ●

d. ▯ < ●

e. ● > ■ ■

f. ■ ■ < ● ●

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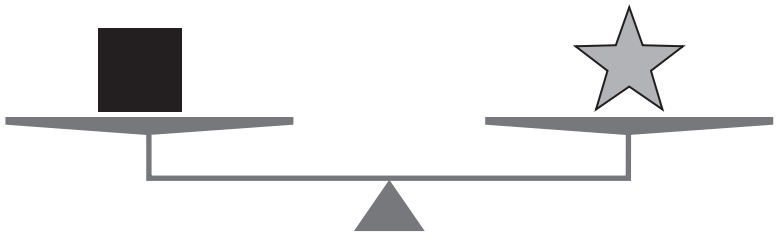

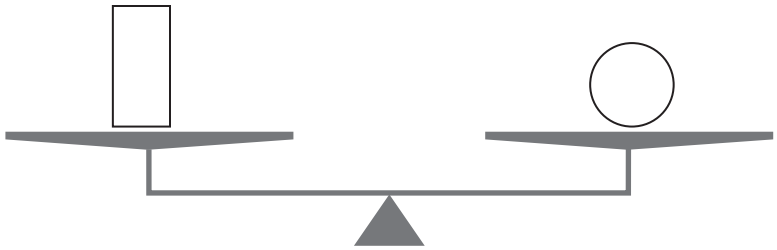

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c. 

d. 

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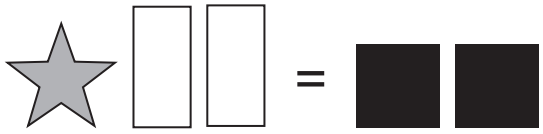
f. 

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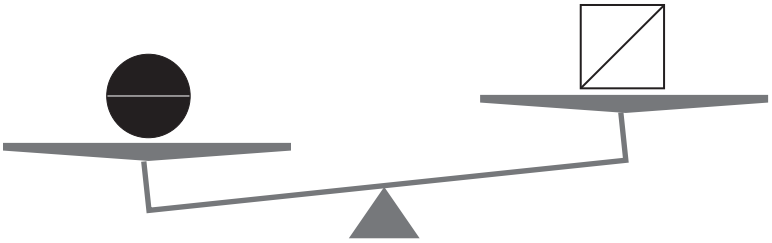

Circle the three answers that will always be true.

a. ● □ > □ □

b. □ □ □ > ●

c. ● ● > □ □

d. ● ▮ < □ ▮

e. □ ● > ● ●

f. ◡ > ◺

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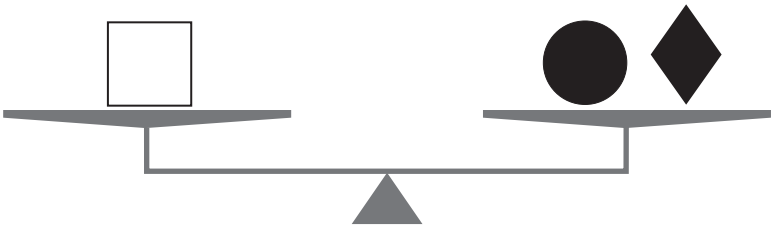

Circle the three answers that will always be true.

a. □ < ◆

b. □ > ●

c. □ □ > ● ● ◆

d. ◆ □ = ●

e. ◆ ● = □ ●

f. ◆ < □

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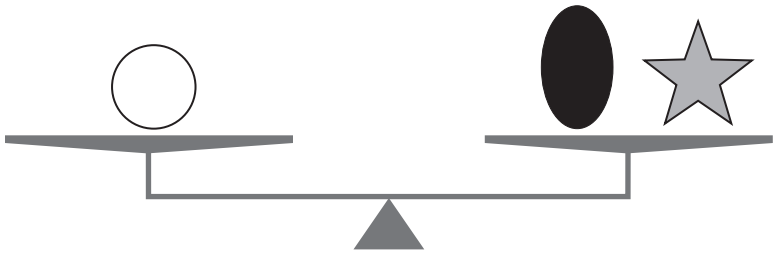

Circle the three answers that will always be true.

a. ○ < ★

b. ★ < ○

c. ⬮ < ○

d. ○ > ⬮

e. ★ = ⬮

f. ⬮⬮ > ○

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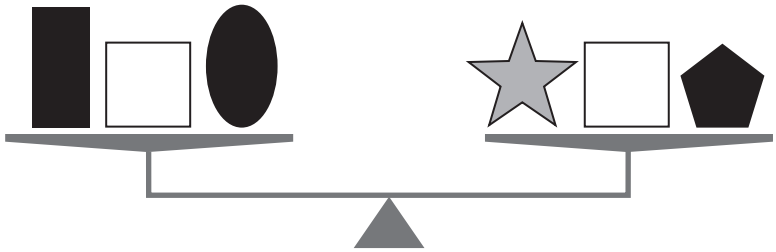

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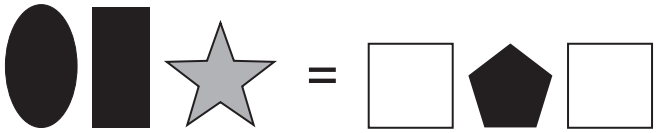
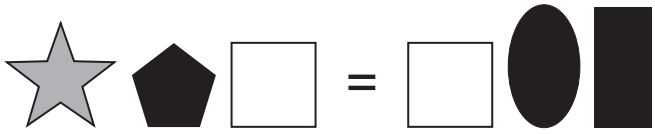
a. 

b. 

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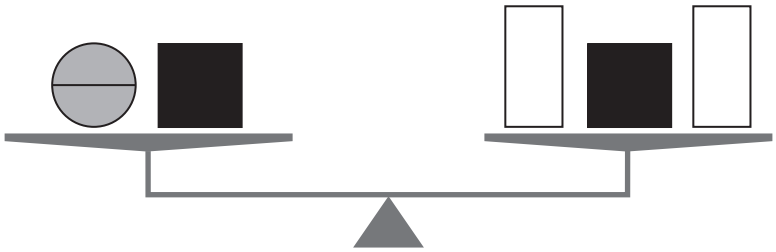

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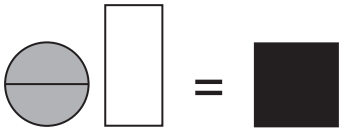
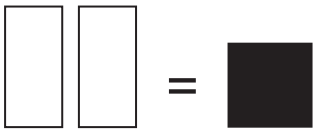
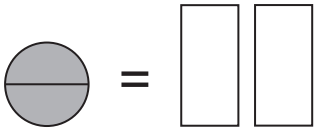
a. 

b. 

c. 

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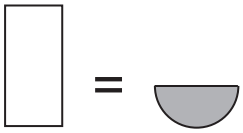
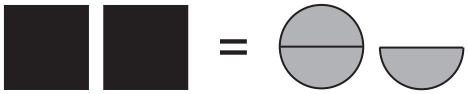
e. 

f. 

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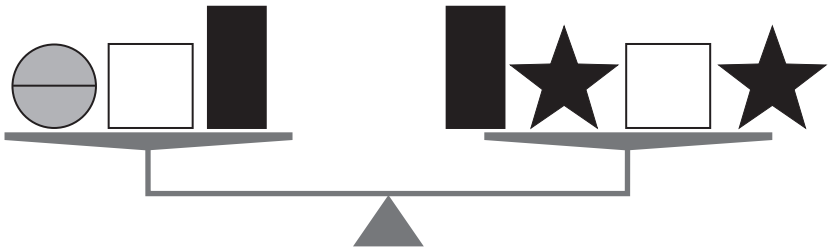

Circle the three answers that will always be true.

a. ★ ★ = ●

b. □ ★ ★ = ● ▮

c. ● □ ★ ▮ ★ = ● ▮ □ ●

d. ★ = □

e. ★ ★ ★ ★ = ●

f. ★ ★ ★ = ● ◡

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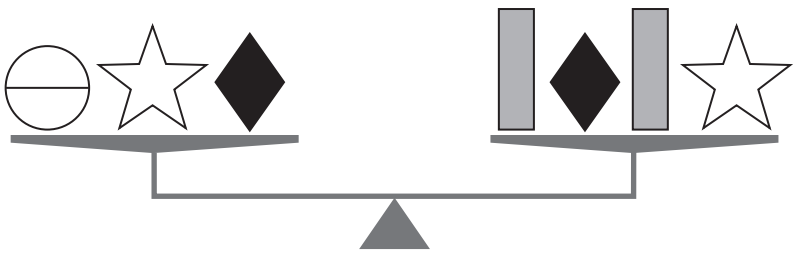

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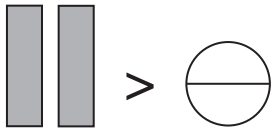
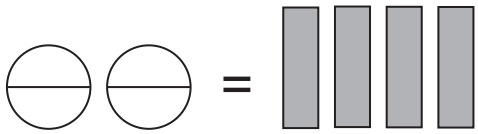
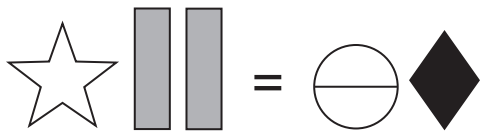
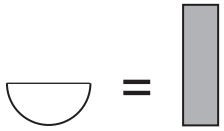
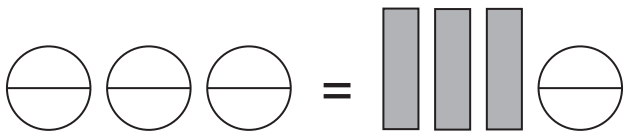
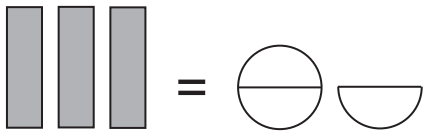
a. 

b. 

c. 

d. 

e. 

f. 

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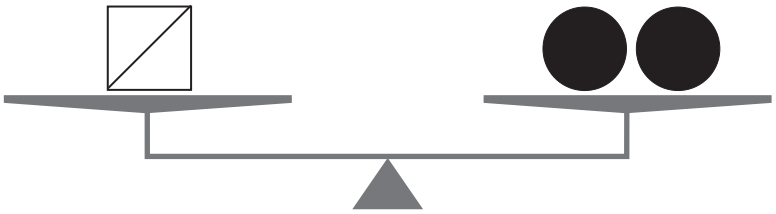


Circle the three answers that will always be true.

a. ● ● < □ ▮▮

b. ● = ◿

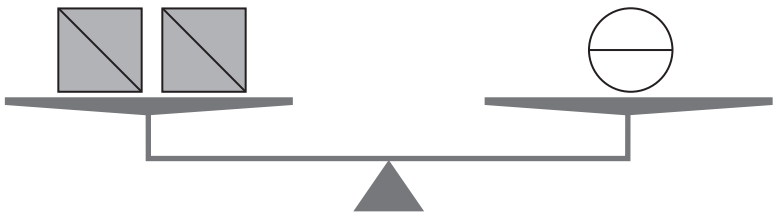
c. □ □ = ● ● ●

d. ● > □

e. ● ● ● = □ ●

f. □ □ < ● ● ●

Balance Benders™



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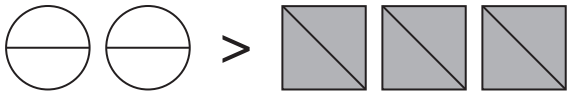
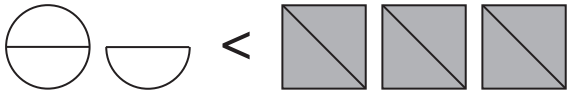
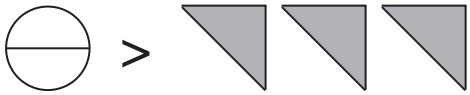
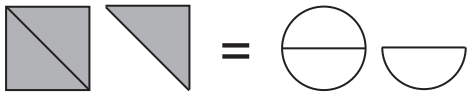
a. 

b. 

c. 

d. 

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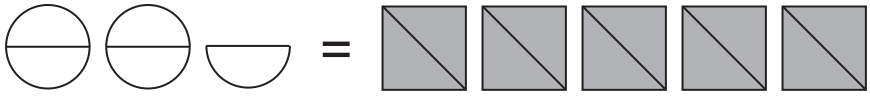
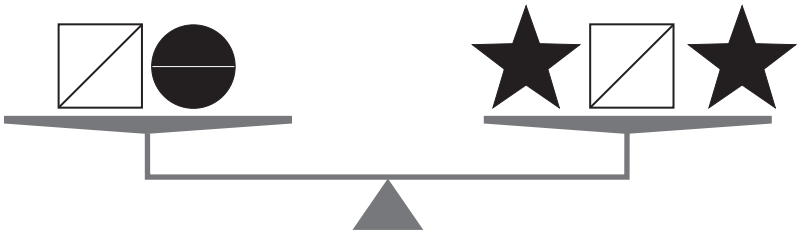
f. 

Balance Benders™



Circle the three answers that will always be true.

a. ★ ★ ◆ = ● ◆

b. ● = ▨

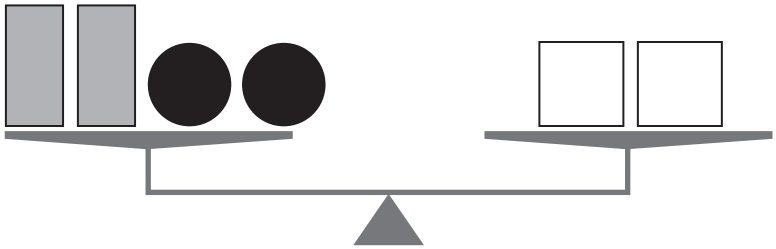
c. ◗ = ★

d. ▨ = ★ ★

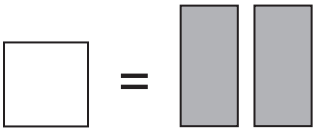
e. ★ ★ ▨ = ● ▨ ●

f. ★ ◗ = ●

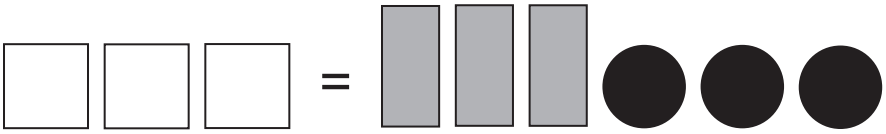
Balance Benders™

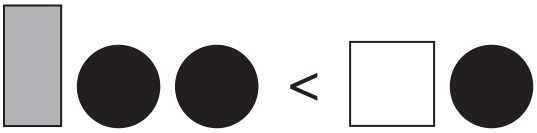


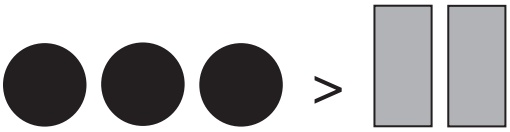
Circle the three answers that will always be true.

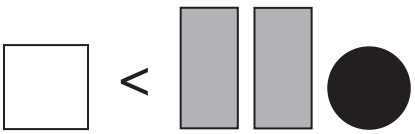
a. 

b. 

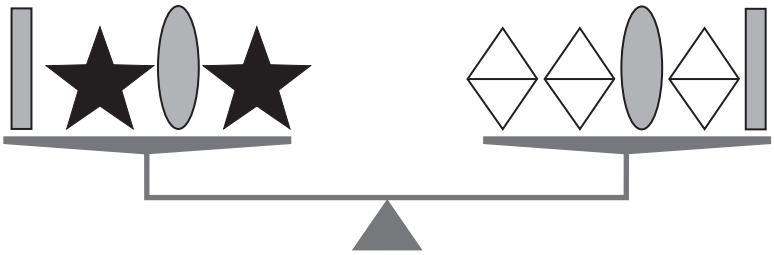
c. 

d. 

e. 

f. 

Balance Benders™



Circle the three answers that will always be true.

a. ◇ < ★

b. ★ = ▽ ▽ ▽

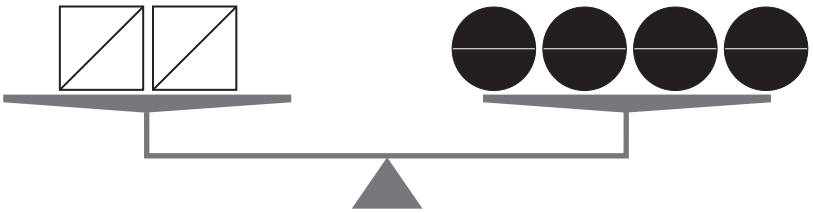
c. ★ ★ ★ < ◇ ◇ ◇ ◇

d. ◇ ◇ = ★◤

e. ★ ★ ★ = ◇ ◇ ◇ ◇ ◇ ◇

f. ★ ▮● ▮ < ▮ ◇ ◇ ●▮

Balance Benders™



Circle the three answers that will always be true.

a. 

b. 

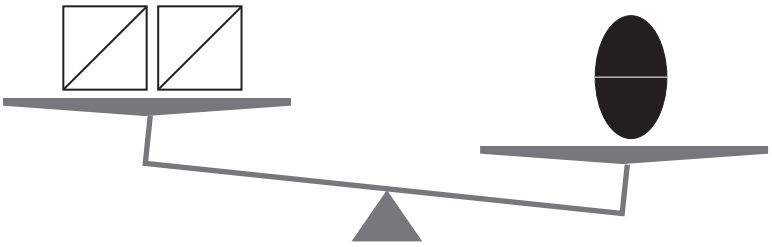
c. 

d. 

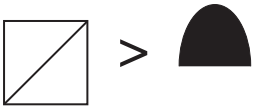
e. 

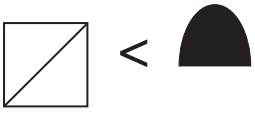
f. 

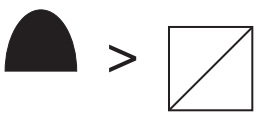
Balance Benders™

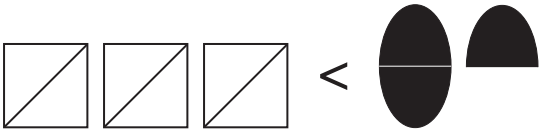


Circle the three answers that will always be true.

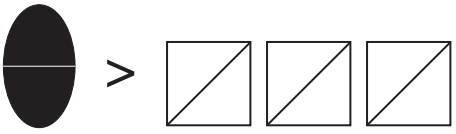
a. 

b. 

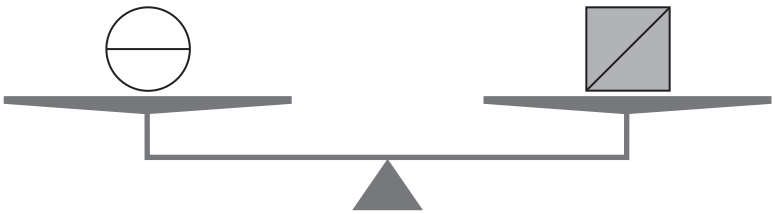
c. 

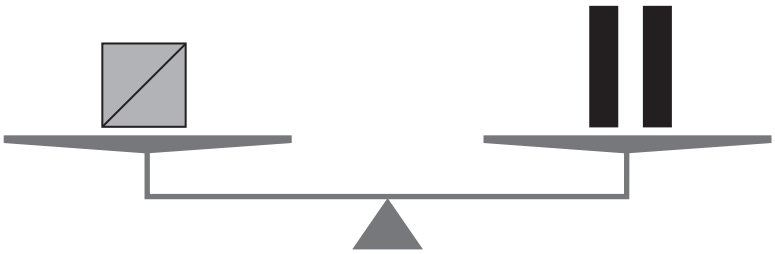
d. 

e. 

f. 

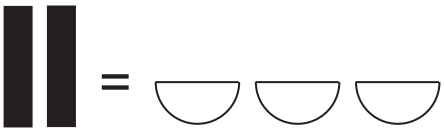
Balance Benders™

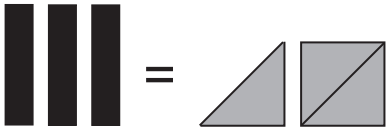


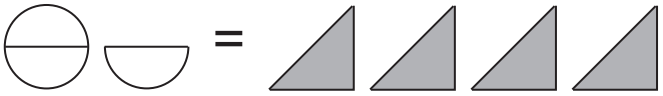


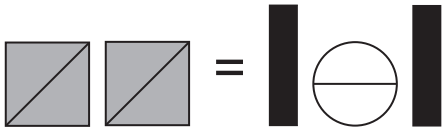
Circle the three answers that will always be true.

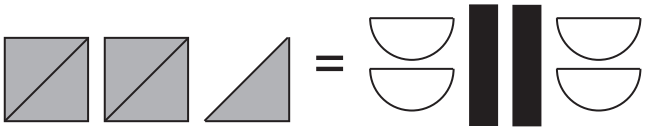
a. 

d. 

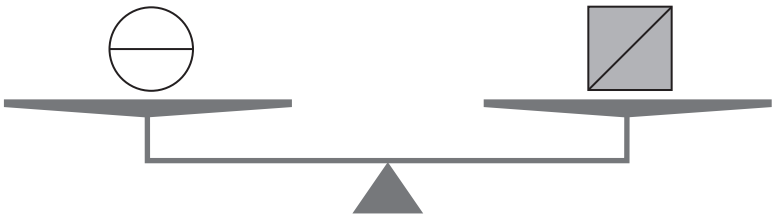
b. 

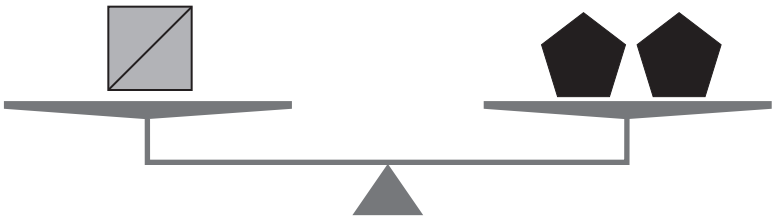
e. 

c. 

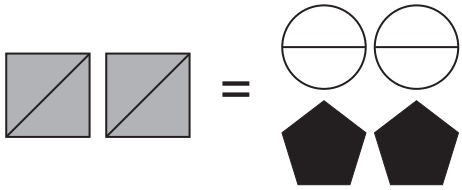
f. 

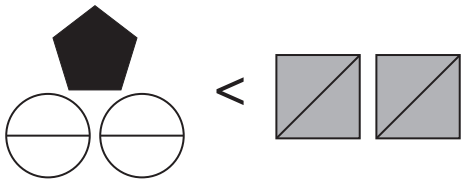
Balance Benders™





Circle the three answers that will always be true.

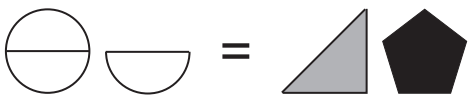
a. 

b. 

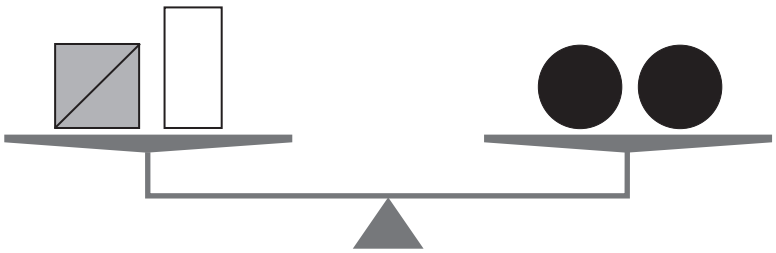
c. 

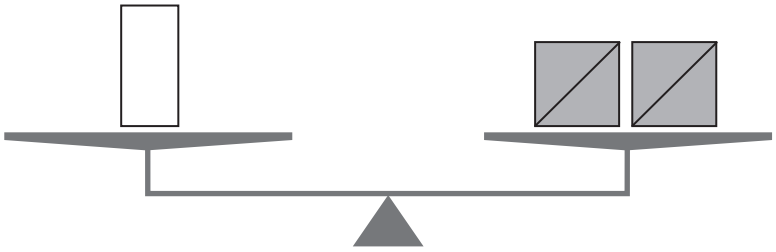
d. 

e. 

f. 

Balance Benders™





Circle the three answers that will always be true.

a. ● ● = ▨ ▨ ▨

b. ▯ > ▨

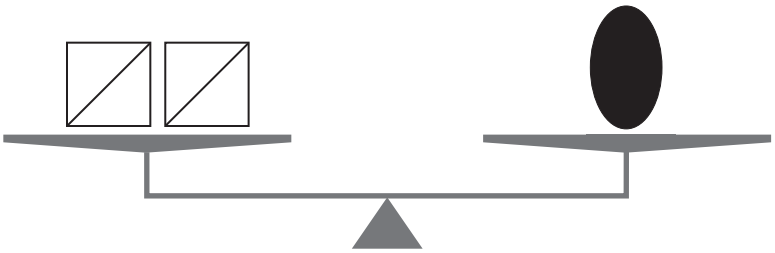
c. ▨ > ▯

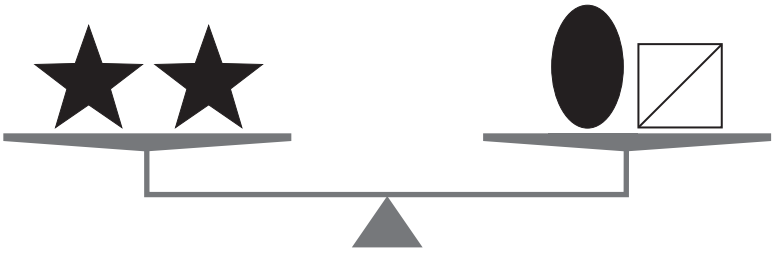
d. ● < ▨

e. ▨ < ●

f. ● > ▨ ◢

Balance Benders™





Circle the three answers that will always be true.

a. ★ = ◿ ◿ ◿

b. ★ ★ > ▭ ▭

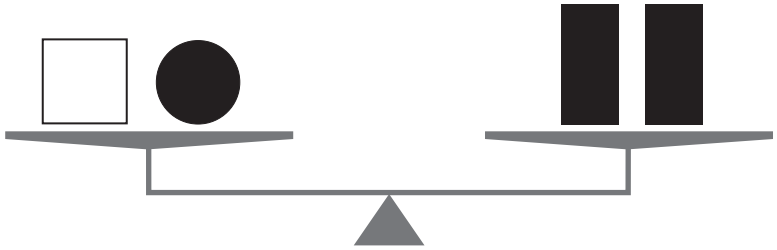
c. ▭ ◿ ★ = ▭ ▭

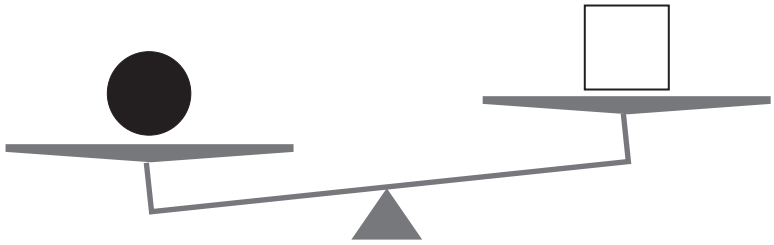
d. ▭ > ★

e. ★ ★ > ● ●

f. ● < ★

Balance Benders™





Circle the three answers that will always be true.

a. ● > □

b. ● ● > ▮ ▮

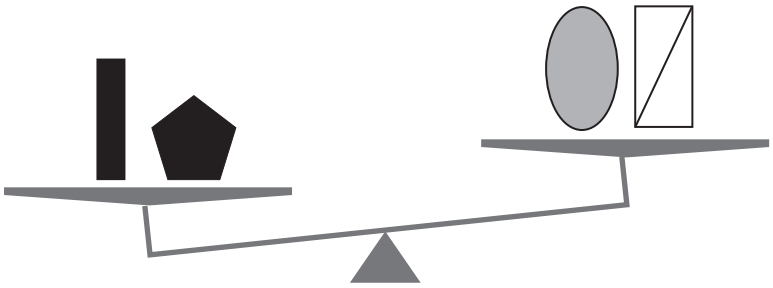
c. □ > ▮

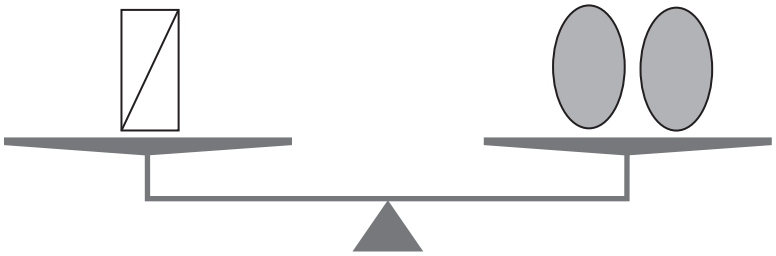
d. ● < □

e. □ □ < ▮ ▮

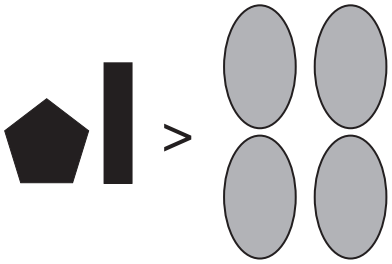
f. □ □ > ●

Balance Benders™

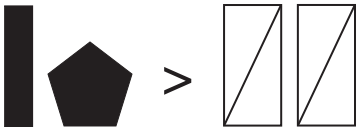




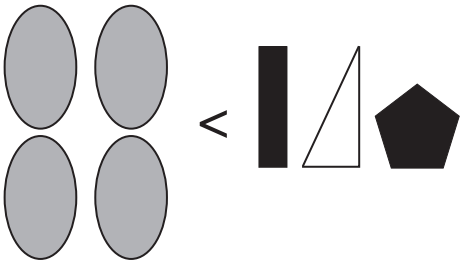
Circle the three answers that will always be true.

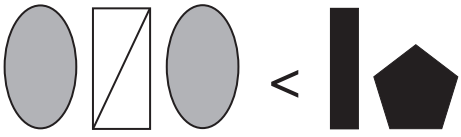
a. 

b. 

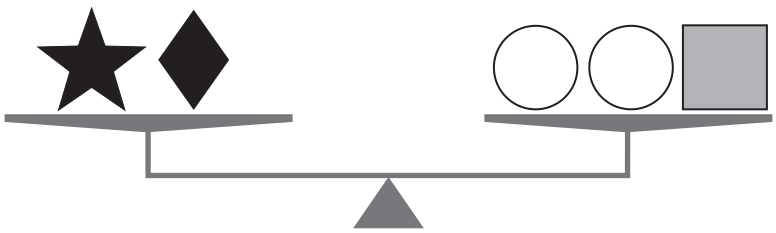
c. 

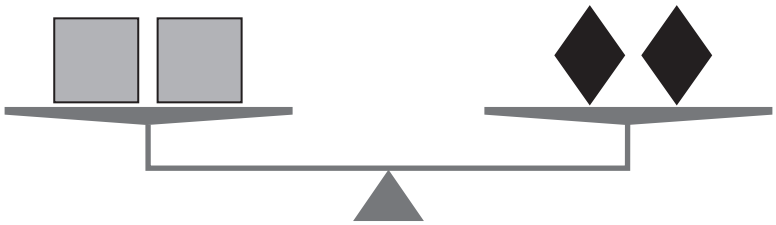
d. 

e. 

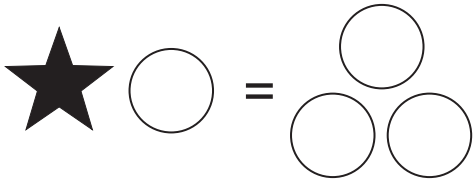
f. 

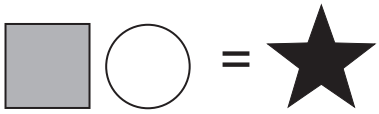
Balance Benders™

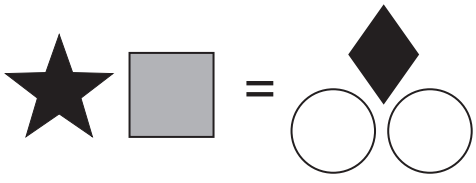




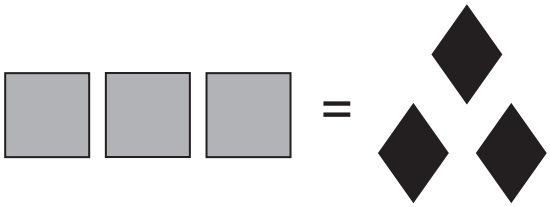
Circle the three answers that will always be true.

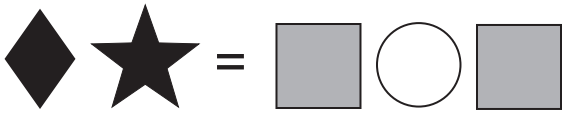
a. 

b. 

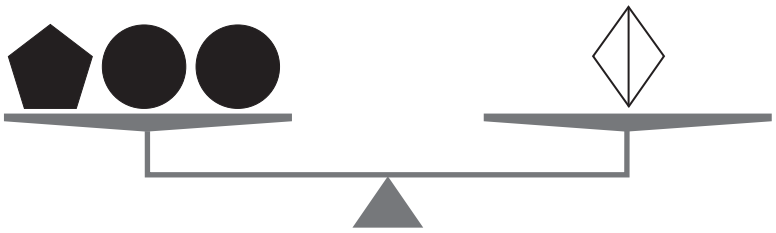
c. 

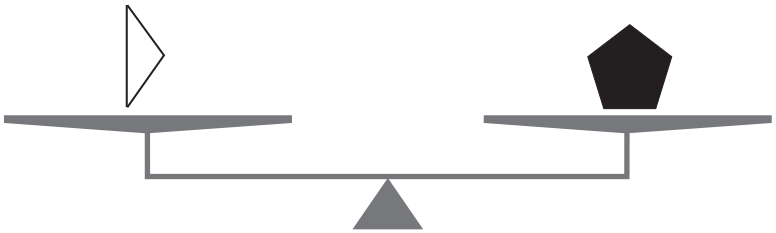
d. 

e. 

f. 

Balance Benders™





Circle the three answers that will always be true.

a. 

b. 

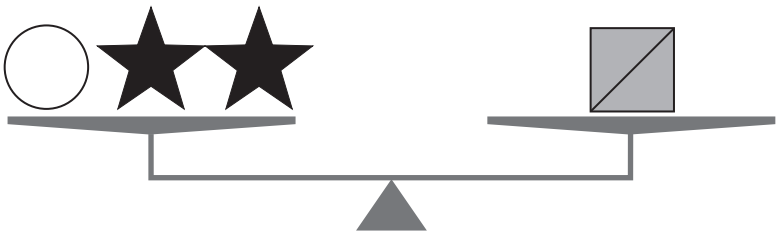
c. 

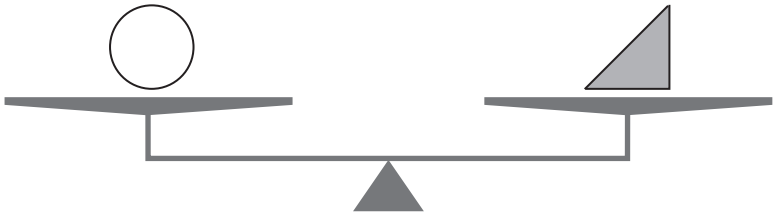
d. 

e. 

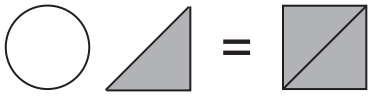
f. 

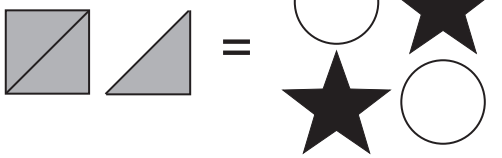
Balance Benders™

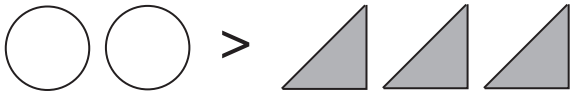


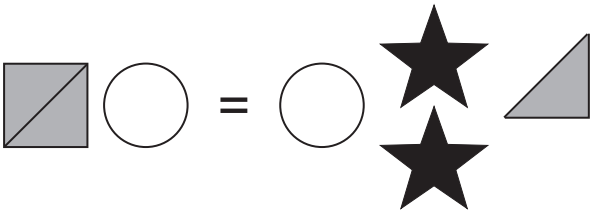


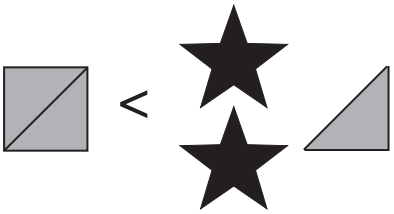
Circle the three answers that will always be true.

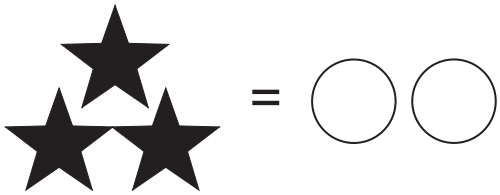
a. 

b. 

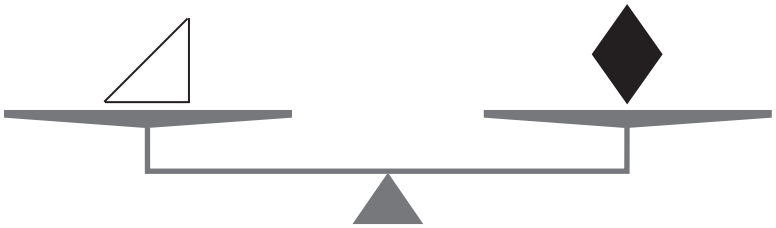
c. 

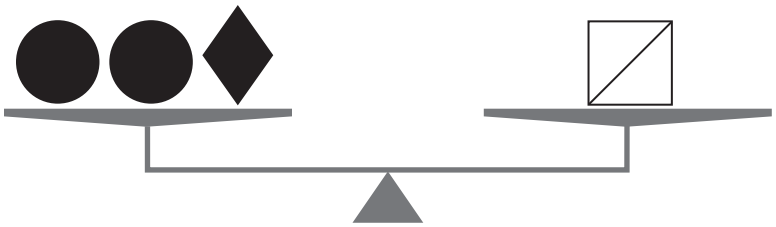
d. 

e. 

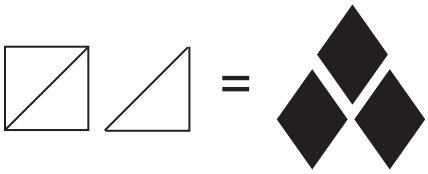
f. 

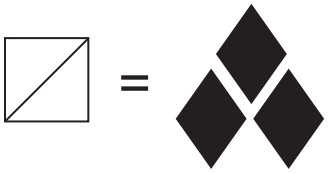
Balance Benders™

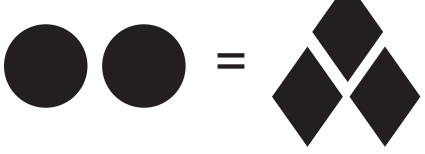




Circle the three answers that will always be true.

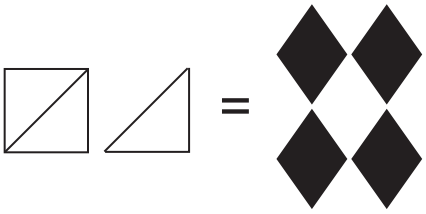
a. 

b. 

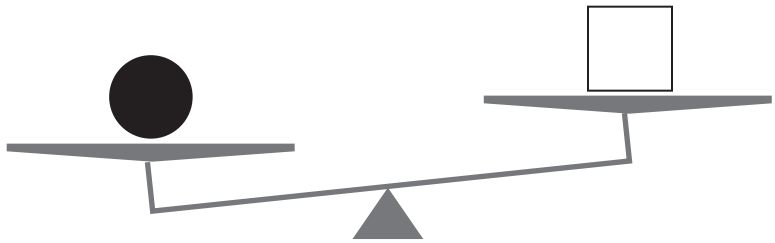
c. 

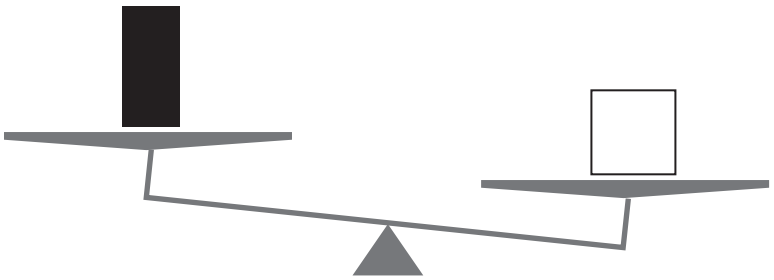
d. 

e. 

f. 

Balance Benders™





Circle the three answers that will always be true.

a. □ > ●

b. ● > □

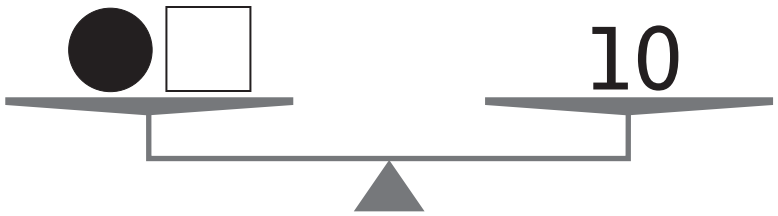
c. □ < ●

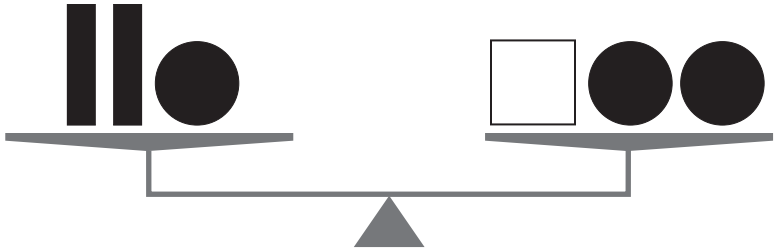
d. □ < ▮

e. ● > □ > ▮

f. ▮▮ > ●

Balance Benders™





Circle the three answers that will always be true.

a. ● = 4

b. □ > ●

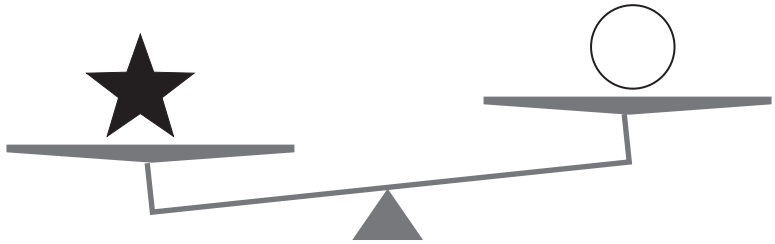
c. ▮▮ = ●□

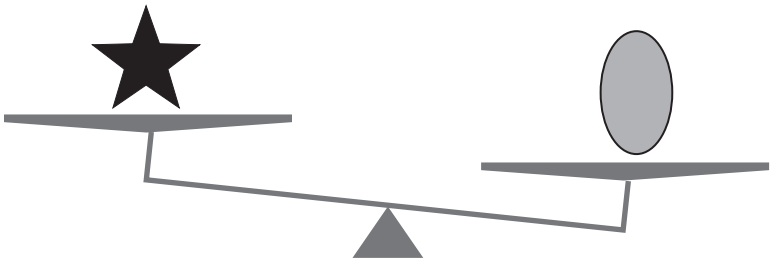
d. ● < ▮

e. ▮ = 5

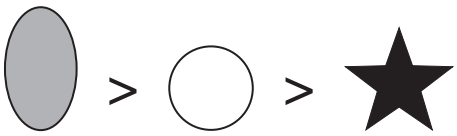
f. □●▮ = 15

Balance Benders™





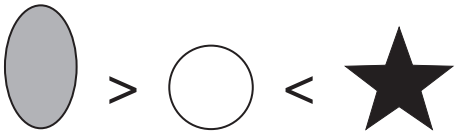
Circle the three answers that will always be true.

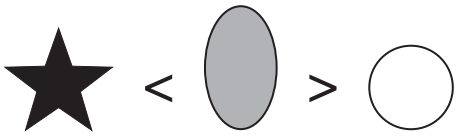
a. 

b. 

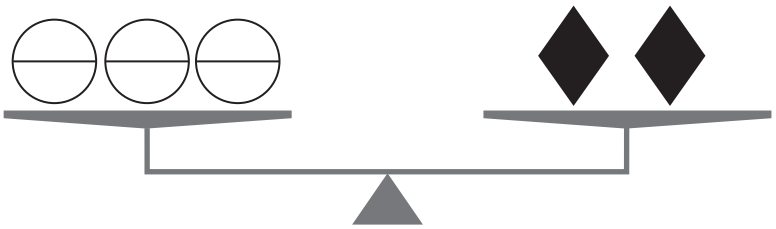
c. 

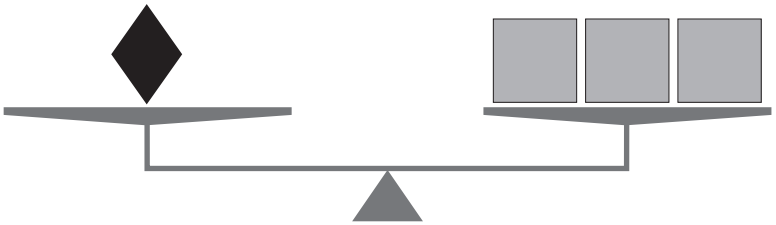
d. 

e. 

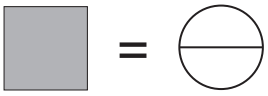
f. 

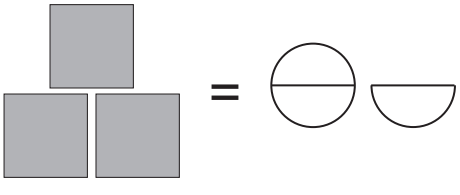
Balance Benders™

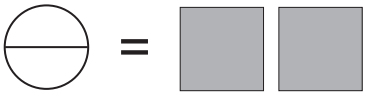


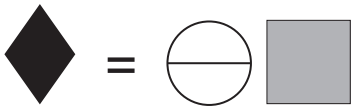


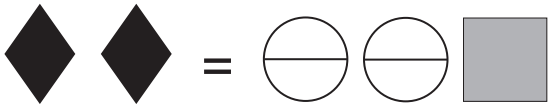
Circle the three answers that will always be true.

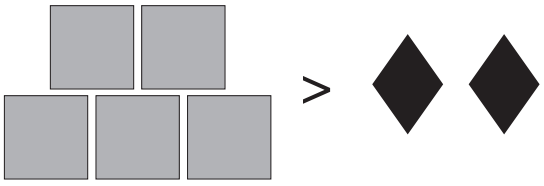
a. 

b. 

c. 

d. 

e. 

f. 

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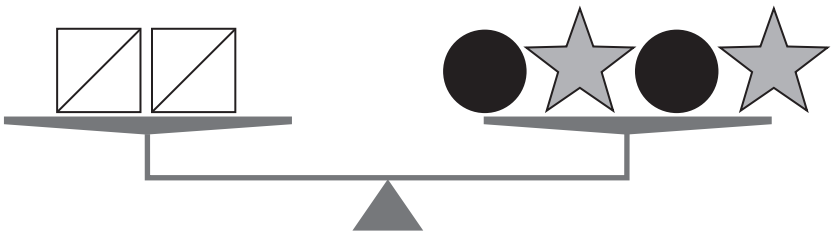
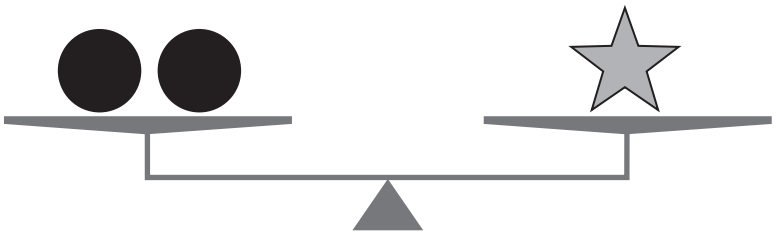


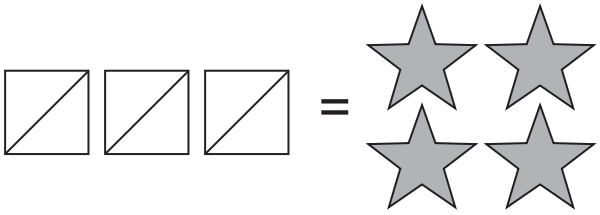


Circle the three answers that will always be true.

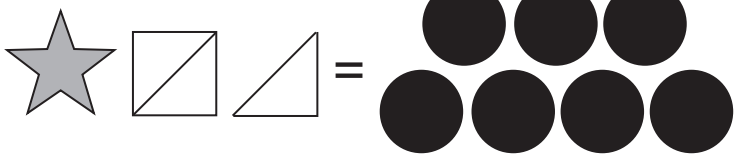
a. 

b. 

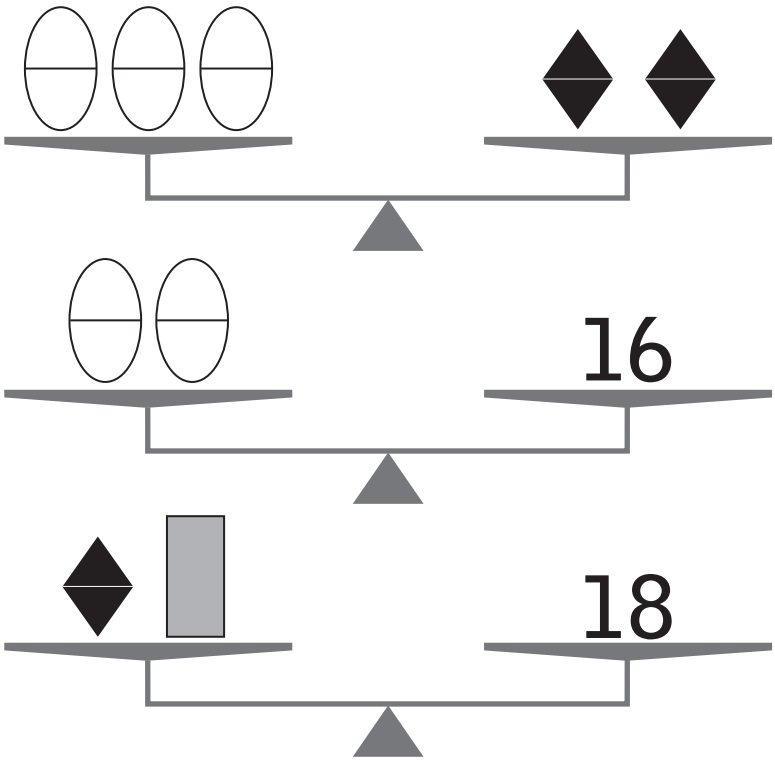
c. 

d. 

e. 

f. 

Balance Benders™



Circle the three answers that will always be true.

a. ♦ = 8

b. ♦▼ = 18

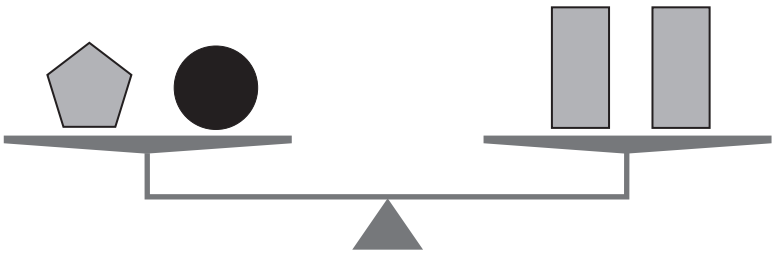
c. ▮▮▮ > 21

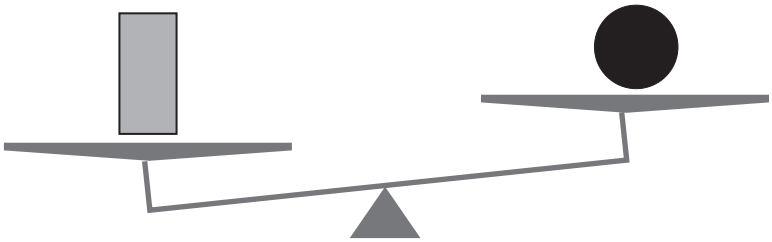
d. ◡ = ▮▮

e. ♦▮ < 25

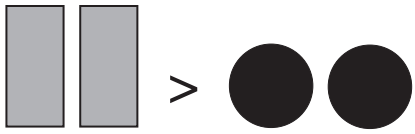
f. ♦▮ > 25

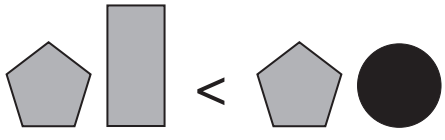
Balance Benders™

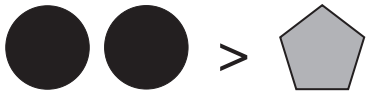


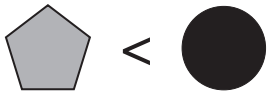


Circle the three answers that will always be true.

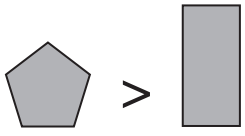
a. 

b. 

c. 

d. 

e. 

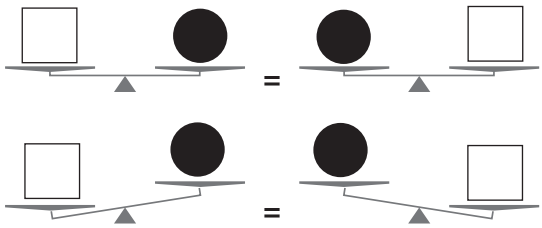
f. 

Balance Tips
(Algebra Concepts)

1. Reversing the pans does not change the balance of the scale. For example:

   If $a = b$ then $b = a$
   If $a > b$ then $b < a$

   Symmetric Property of Equality and Inequality



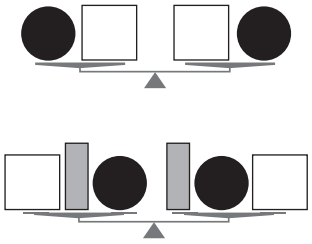
2. Rearranging "weights" does not change the balance of the scale. For example:

   $a + b = b + a$

   Commutative Property of Equality

   $a + (b + c) = (a + b) + c$

   Associative Property of Equality



3. Adding the same "weight" to each pan does not change the balance of the scale. For example:

   If $a = b$ then $a + c = b + c$

   Addition Property of Equality

4. Subtracting the same "weight" from each pan does not change the balance of the scale. For example:

   If $a = b$ then $a - c = b - c$

   Subtraction Property of Equality

5. Multiplying both pans equally (e.g. doubling) does not change the balance of the scale. For example:

   If $a = b$ then $a \cdot c = b \cdot c$

   Multiplication Property of Equality

Balance Tips (Cont.)
(Algebra Concepts)

6. Dividing or partitioning both pans into equally numbered groups (e.g. take half) does not change the balance of the scale. For example:

If $a = b$ then $a/c = b/c$

Division Property of Equality

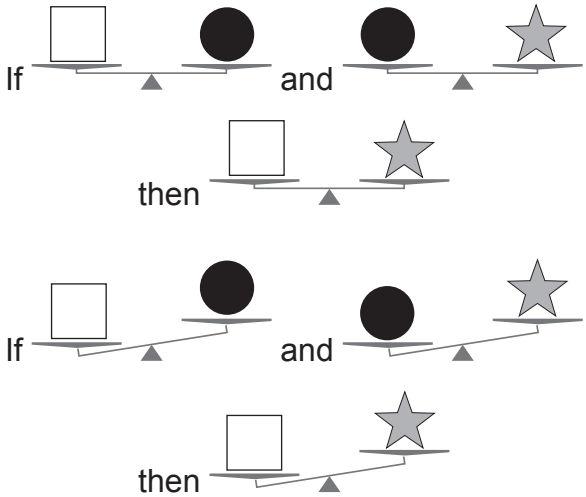
7. Substitute one "weight" for a similar "weight" or group of "weights". For example:

If $a = b$ then "a" can be substituted for "b" in any equation or inequality

Substitution Property of Equality and Inequality

If $a = b$ and $b = c$ then $a = c$
If $a < b$ and $b < c$ then $a < c$
If $a > b$ and $b > c$ then $a > c$

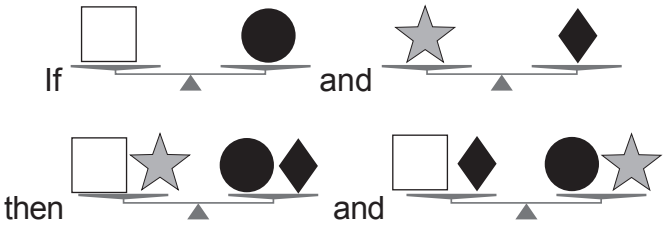
Transitive Property of Equality and Inequality



8. Combining two balanced scales does not change the balance of the new scale. For example:

If $a = b$ and $c = d$ then $a + c = b + d$ and $a + d = b + c$

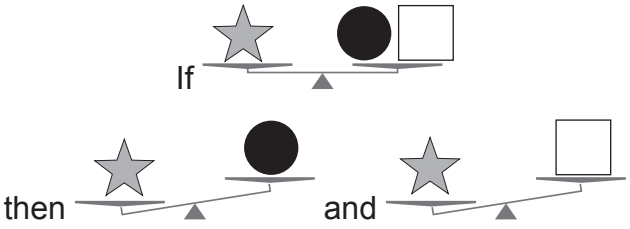
Addition and Substitution Properties



9. Removing a "weight" from one pan of a balanced scale causes an imbalance. For example:

If $a + b = c$ then $c > a$ and $c > b$

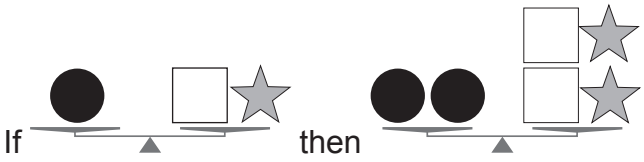
Equation to Inequality or Trichotomy Property



10. When multiplying or dividing, be sure to do the same to all "weights" in the pans. For example:

$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

Distributive Property



Solutions

Page 1: a, c, e

a. Reversing the pans does not change the balance. (Tip 1)

c. Add  to each pan. (Tip 3)

e. Double both pans then reverse. (Tips 5 and 1)

Page 2: c, d, e

c. Add  to both pans. (Tips 3 and 1)

d. Add  to both pans then reverse. (Tips 3 and 1)

e. Add  to both pans then reverse. (Tips 3 and 1)

Page 3: b, e, f

b. Add  to both pans. (Tip 3)

e. Add  to both pans. (Tip 3)

f. Add  to both pans. (Tip 3)

Page 4: a, b, d

a. Two half squares make one whole square.

b. Two half squares make one whole square.

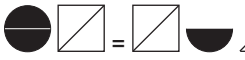
d. Two half circles make one whole circle.

Page 5: c, d, e

c. Double both pans, then reverse. (Tips 5 and 1)

d. Both shapes are split in half but the balance does not change. (Tip 4)

e. Add  to both pans so  (Tip 3)

Substitute a half circle for a half square, since  so  (Tip 7)

Page 6: c, d, f

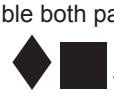
c.  is heavier than  so  .

d.  is lighter than  so  .

f. Doubling both pans does not change the balance. (Tip 5)

Page 7: c, d, e

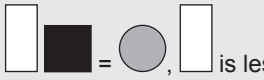
c. Double both pans then reverse. (Tips 5 and 1)

d. Add  to both pans. (Tip 3)

e. If  , then  is heavier than  .

Page 8: c, d, f

c. Since  , is less than  . (Tip 9)

d. Since  , is less than  . (Tip 9)

f. Since  , doubling both sides does not change the balance, so  . (Tip 5)

Page 9: b, d, e

b. Reverse 1st balance, then add both balances together. (Tip 8)

d. Add both balances together. (Tip 8)

e. Remove the white rectangles from each side. (Tips 4)

Page 10: a, c, f

a. Adding  to both pans does not change the balance. (Tip 3)

c. Doubling both pans does not change the balance. (Tip 5)

f. Dividing both pans in half does not change the balance. (Tip 6)

Page 11: b, c, f

b. Remove  from only one pan so  . (Tip 9)

c. Double both pans  (Tip 5)

so  . (Tip 9)

f. Remove  from only one pan so  . (Tip 9)

Page 12: b, c, d

b. Remove  so  . (Tip 9)

c. Remove  so  . (Tip 9)

d. Remove so . (Tip 9)

Page 13: b, e, f

b. Rearrange pans. (Tip 2) Reverse. (Tip 1)

e. Remove from both pans. (Tip 4)

f. Add to both side and reverse. (Tip 3)

Page 14: c, d, e

c. Remove  from both pans. (Tip 4)

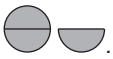
d. Add  to both pans and reverse. (Tips 3 and 1)

e. Divide  =  in half so  =  . (Tip 6)

Page 15: a, c, f

a. Remove  and  from both pans. (Tip 4)

c. Add  to both pans. (Tip 3) Reverse pans. (Tip 1) Rearrange. (Tip 2)

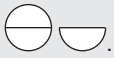
f. If  =  , then  =  and  =  =  . (Tip 6)

Page 16: b, d, f

Remove  from both pans so  =  . (Tip 4)

b. Double both pans so  =  . (Tip 5)

d. Divide both pans in half so =  . (Tip 6)

f. If  =  and  =  , then  =  .

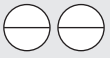
Page 17: a, b, e

a. Add  to one pan and  to the other pan so  . (Tip 9)

b. Divide each pan in half and reverse. (Tips 6 and 1)

e. Add  to both pans and reverse. (Tips 3 and 1)

Page 18: a, c, f

a. Double both pans so  =  . (Tip 5) Remove  so  >  . (Tip 9)

c.  =  = . Remove so > . (Tip 9)

f. Divide both pans in half so = . (Tip 6) Multiply by 5 so = . (Tip 5)

Page 19: a, c, f

Remove from both pans so . (Tip 4)

a. Add to both pans. (Tip 3) Reverse pans. (Tip 1)

c. Divide in half so . (Tip 6)

f. Substitute for in so = . (Tip 7) Reverse pans and rearrange. (Tips 1 and 2)

Page 20: b, c, f

b. Divide both pans in half. (Tip 6)

c. Add to both pans, then substitute for on left pan. (Tips 3 and 7)

f. must weigh less than . Since = , weighs less than . (Tip 7)

Page 21: a, b, f

Remove and from both pans so = .

(Tip 4) Divide both pans in half so = . (Tip 6)

a. Therefore < . (Tip 9)

b. = , so = .

f. Remove from both sides so < , which is true since = . (Tip 4)

Page 22: b, e, f

Divide both pans in half so = . (Tip 5)

b. Add to both sides of = so = . (Tip 3)

e. Substitute for . = . (Tip 7)

f. If = , then = . (Tip 5) Therefore > . (Tip 9)

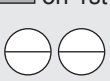
Page 23: b, c, d

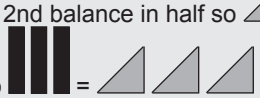
b. Divide both pans in half so < . (Tip 6)

c. If < , then > . (Tip 1)

d.  (Tip 5)

Page 24: a, b, c

a. Substitute  from 2nd balance for  on 1st balance so  . (Tip 7) Double both pans  . (Tip 6) Reverse pans. (Tip 1)

b. Divide 2nd balance in half so  . (Tip 6) Triple each pan so  . (Tip 5)

c. Add both scales together. (Tip 8) Rearrange so  . (Tip 2)

Page 25: c, d, e

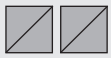
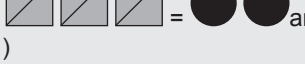
Substitute  from 2nd balance for  on 1st balance so  . (Tip 7)

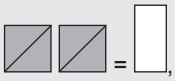
c. Half of both pans is  . (Tip 6)

d. Add  to both pans on 2nd balance. (Tip 4)

e. If  , then  and if  , then  , so  . (Tips 6 and 7)

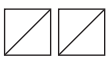
Page 26: a, b, e

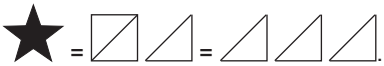
a. Substitute  from 2nd balance for  on 1st balance.  and reverse pans. (Tips 7 and 1)

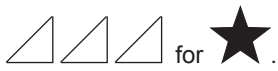
b. If  , then  is heavier than  . (Tip 9)

e.  (Tips 9 and 1)

Page 27: a, c, d

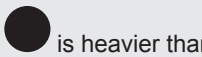
Substitute  from 1st balance for  on 2nd balance so  . (Tip 7)

a. Half of each pan is  (Tip 6)

c. Substitute  for . (Tip 7)

d. Since  , then  .

Page 28: a, b, e

a. 

b. From 1st balance,  . Since  , then  . (Tip 7)

e. From 1st balance,  . (Tip 7)

Page 29: b, d, e

From 2nd balance, substitute  for  on 1st balance. is heavier than .

b.

d. From 2nd balance, . (Tip 5) Substitute for on 1st balance. . (Tip 7)

e. Since , adding the same weigh to both pans does not change the balance. (Tip 3)

Page 30: a, c, e

From 2nd balance, . (Tip 6) Removing and from 1st balance shows .

a. Add to both pans of . (Tip 3)

c. . Adding one to each pan of does not change the balance. (Tip 3)

e. Triple both pans of . (Tip 5)

Page 31: b, c, d

b. Double both pans in 2nd balance. (Tip 5)

c. Substitute for in 1st balance so . (Tip 7) Remove from each pan. (Tip 4)

d. Substitute for . (Tip 7)

Page 32: a, b, d

Double the 2nd balance so ○○ = ▨.

a. Add ◢ to both pans in 2nd balance. (Tip 3)

b. Add the balances together so ○○★★ = ▨◢. Rearrange and reverse pans. (Tip 8)

d. Reverse 1st balance and add together. (Tips 1 and 8)

Page 33: a, d, e

Double 1st balance so ▨ = ♦♦.

a. Triple the 1st balance so ◢◢◢ = ♦♦♦ and ◢◢◢ = ▨◢. (Tip 5)

d. Substitute ♦♦ for ▨ on 2nd balance. (Tip 7) Remove ♦ from both pans.

e. Substitute ◢ for ♦. (Tip 7)

Page 34: b, c, e

● > □ and □ > ▮.

b. Given.

c. (Tip 1)

e. (Tip 7)

Page 35: c, e, f

c. Remove ● from each pan on 2nd balance so ▮▮ = □●. (Tip 4) Rearrange. (Tip 2)

e. Remove ● from each pan on 2nd balance so ▮▮ = □●. (Tip 4) Substitute 10 for □● so ▮▮ = 10. (Tip 7) Divide in half so ▮ = 5. (Tip 6)

f. If □● = 10 and ▮ = 5, then □●▮ = 15.

Page 36: b, e, f

⬭ > ★ and ★ > ○.

b. (Tip 7)

e. (Tip 1)

f. (Tip 1)

Page 37: b, c, d

From the 1st balance, ⊖⌓ = ♦. (Tip 6)

b. Substitute ⊖⌓ for ♦ on 2nd balance. ⊖⌓ = ▭▭▭ or ▭▭▭ = ⊖⌓. (Tip 7)

c. ⊖⌓ = ▭▭▭. Divide both pans into thirds to find ⌓ = ▭ (Tip 6) Double, so ⊖ = ▭▭. (Tip 5)

d. Substitute ▭▭ = ⊖ on 2nd balance. ♦ = ⊖▭. (Tip 7)

Page 38: b, c, e

b. On 1st balance, substitute ★★ for ●● from 2nd balance. Since ▨▨ = ★★★, then ★ < ▨. Substitute ●● for ★ and ●● < ▨. (Tip 8)

c. Substitute ●● for each ★ on 1st balance so ▨▨ = ●●●●●●. (Tip 7) Divide in half so ▨ = ●●●. (Tip 6)

e. Divide ▨▨ = ★★★ in half. (Tip 6)

Page 39: b, d, f

From 2nd balance, ⊖⊖ = 16, so ⊖ = 8. (Tip 6) From 1st balance, ⊖⊖⊖ = 24. Since 24 = ♦♦, then 12 = ♦. From 3rd balance, 12 + ▭ = 18, so ▭ = 6.

b. ♦▼ = 12 + 6 = 18.

d. ⊖ = 8, so ⊖⌓ = 8 + 4 = 12 and ▭▭ = 12.

f. ⊖♦▭ = 8 + 12 + 6 = 26.

Page 40: a, e, f

a. Since ▭ is heavier than ●, then ▭▭ > ●●. (Tip 5)

e. If ▭▭ = ●⬠, then ⬠ > ● or ● < ⬠. (Tip 9)

f. Since ▭▭ = ●⬠ and ● < ⬠, then ⬠ > ▭. (Tip 9)